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Photometry of the Sunspots Continuous Spectrum

by

A. Stankiewicz

SUMMARY

On the basis of observations of sunspots made in the Crimean Astrophysical Observatory, we found for the four large sunspots the intensity distribution of the continuous spectrum between 4500 and 6300 Å as well as the temperature considered as a function of the optical depth. In addition we obtained the coefficients of absorption $\varkappa_\lambda^*/\overline{\varkappa}$ for 5 chosen windows of the continuous spectrum of the sunspots. The gradients of the temperature determined in our case are smaller than those evaluated by Michard [11] but larger than those given by Makita and Morimoto [10].

РЕЗЮМЕ

На основе наблюдений солнечных пятен произведенных нами в Крымской Астрофизической Обсерватории мы получили распределение непрерывного спектра солнечных пятен в пределах длин волн от 4500 до 6300 А а также распределение температуры с оптической глубиной для четырех больших пятен. Кроме того мы подсчитали коэффициэнты поглощения $\kappa_\lambda/\overline{\kappa}$ для пяти участков непрерывного спектра-на блюдаемых пятен. Градиэнты температуры в нашем случае получились меньше чем у Мишарда [11] но больше чем у Макита и Моримото [10].

1. Introduction

The principal aim of the photometry of the continuous spectrum of sunspots is to find the intensity distribution as a function of the wave length and to determine the limb darkening laws. For this purpose it is necessary to measure the intensity ratios of the spot and the photosphere for various wave lengths and in different points of the solar disc. The differences between the intensity ratios found by various authors [1, 7, 8, 10, 11, 14, 15, 17] correspond to different physical conditions in the sunspots and cannot be treated as observation errors. For this reason a statistical investigation of photometrical measurements of sunspots is to be regarded as an approximation and each sunspot should be, as a rule, treated individually.

The aim of the present paper is to determine the continuous spectrum and the temperature distribution as a function of the optical depth for a few chosen sunspots. To reduce the observational errors

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as much as possible we limited our observations to the case of large sunspots in the visual part of the spectrum. The observations were made in the best possible atmospherical conditions.

2. Observations

The observations were carried out in the Crimean Astrophysical Observatory by using a grating spectrograph and the solar tower telescope [16]. The diameter of the image of the Sun on the spectrograph slide was equal to 320 millimeters. The dispersion in the IV order spectrum was 0·3 Å per millimeter. The following windows of the continuous spectrum were chosen to be measured:

$$\lambda\lambda \quad 6305 \quad 6172 \quad 5596 \quad 5310 \quad \text{and } 4500 \text{ Å}.$$

To get the best photometrical results sensitometric investigations on many photographic emulsions were carried out in conditions where the Eberhard effect was not too great. The gradation of the emulsions was chosen in such a manner that the densities corresponding to the center of the sunspots and the photosphere are either in the linear part of the calibration curve or in its near neighbourhood. Adequate results were obtained on Ilford HP3 plates by using the developer of D76d type. The plate calibrations were obtained by a 9-step platinum filter of known transparency.

A few hundred spectrogramms for several sunspots were taken. Besides the sunspots spectra the spectra of the solar limb were photographed on the same plate simultaneously to determine the corrections of the influence of the scattered light and the atmospheric turbulence. The photographic material obtained in this way was measured by using a recording microphotometer MF4 for chosen wave lengths. As a result of the analysis of sun limb records the material was selected with regard to the quality of the image. About 90 spectrogramms were chosen for investigation; the corresponding limb profile records indicated a very small diffusion caused by atmospheric oscillations. Then the calibration curves were constructed for these spectrogramms and a photometrical profile of sunspots i. e. the values $I_\lambda^S(0\vartheta)/I_\lambda^P(0\vartheta)$ for points situated along a chosen cross-section of a spot, were calculated. These profiles were obtained for sunspots lying consecutively in different distances from the center of the sun's disc. After allowance for perspective foreshortening the following two corrections were introduced: for light scattering and atmospherical turbulence. Assuming that the functions of intensity redistribution have the form given by Wanders [18]

$$\Psi = \frac{a}{\pi} e^{-a(x^2+y^2)}$$

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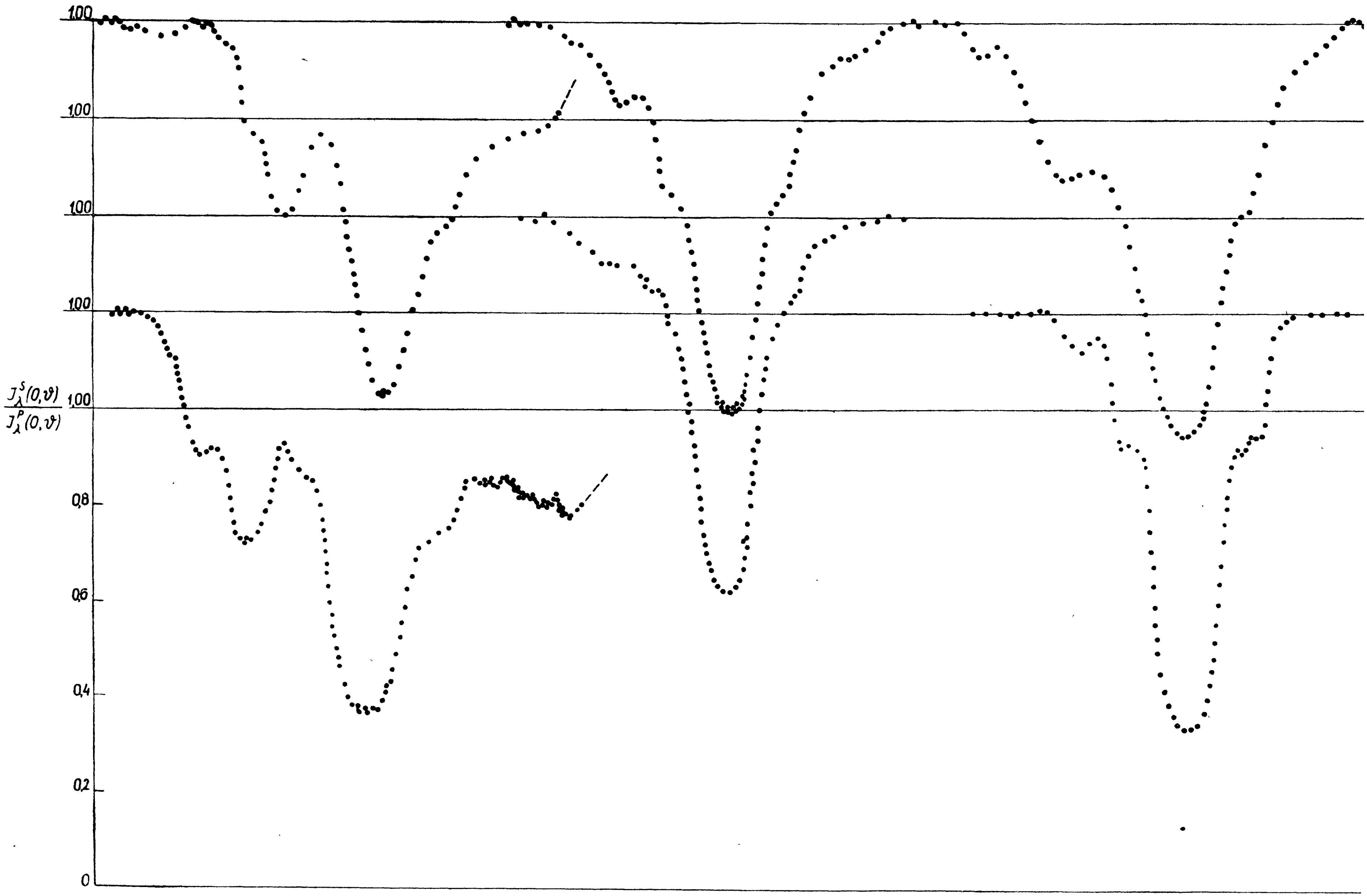

Fig. 1. Photometrical profiles for suns]

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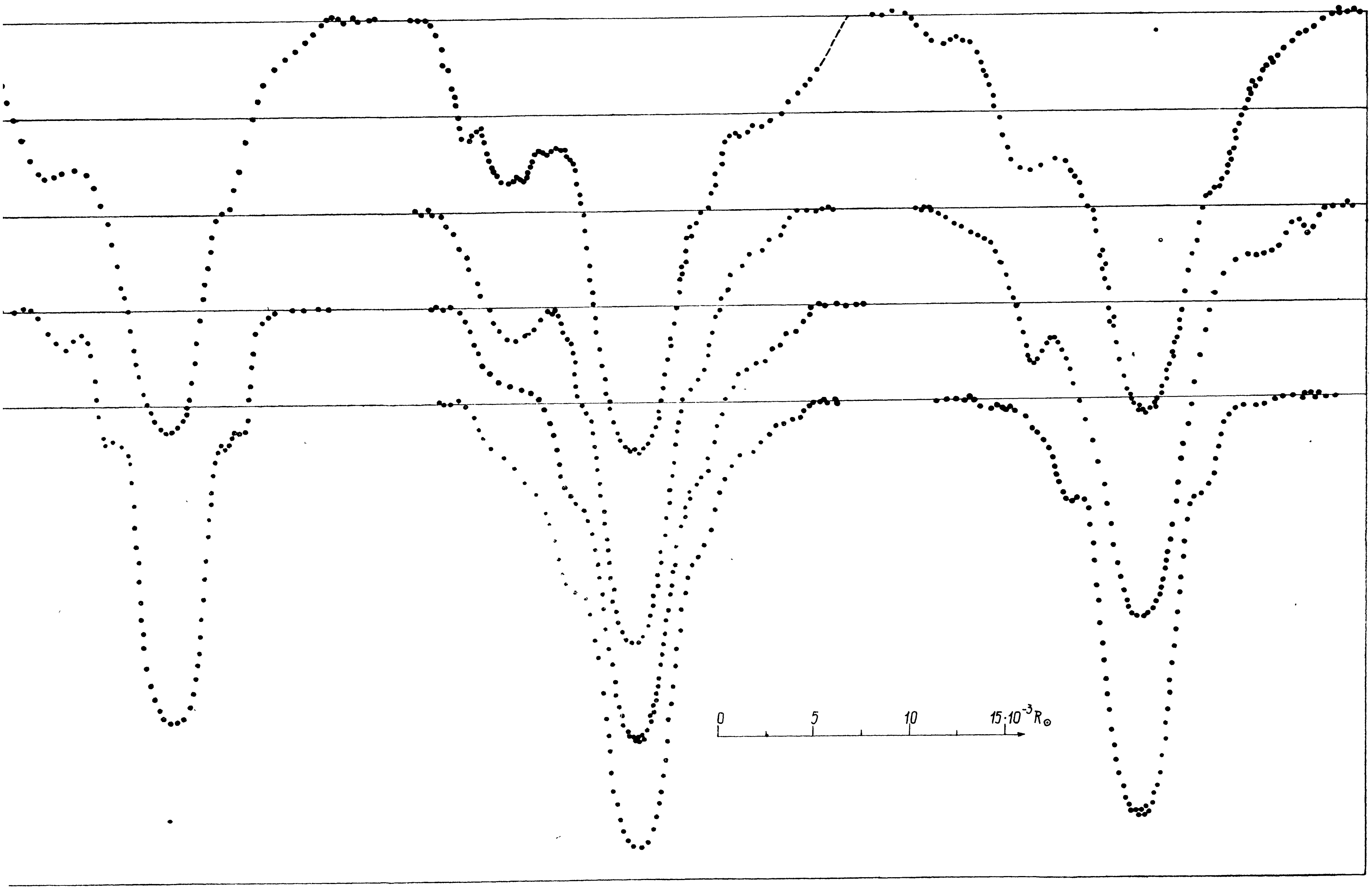

Fig. 1. Photometrical profiles for sunspot No 535.

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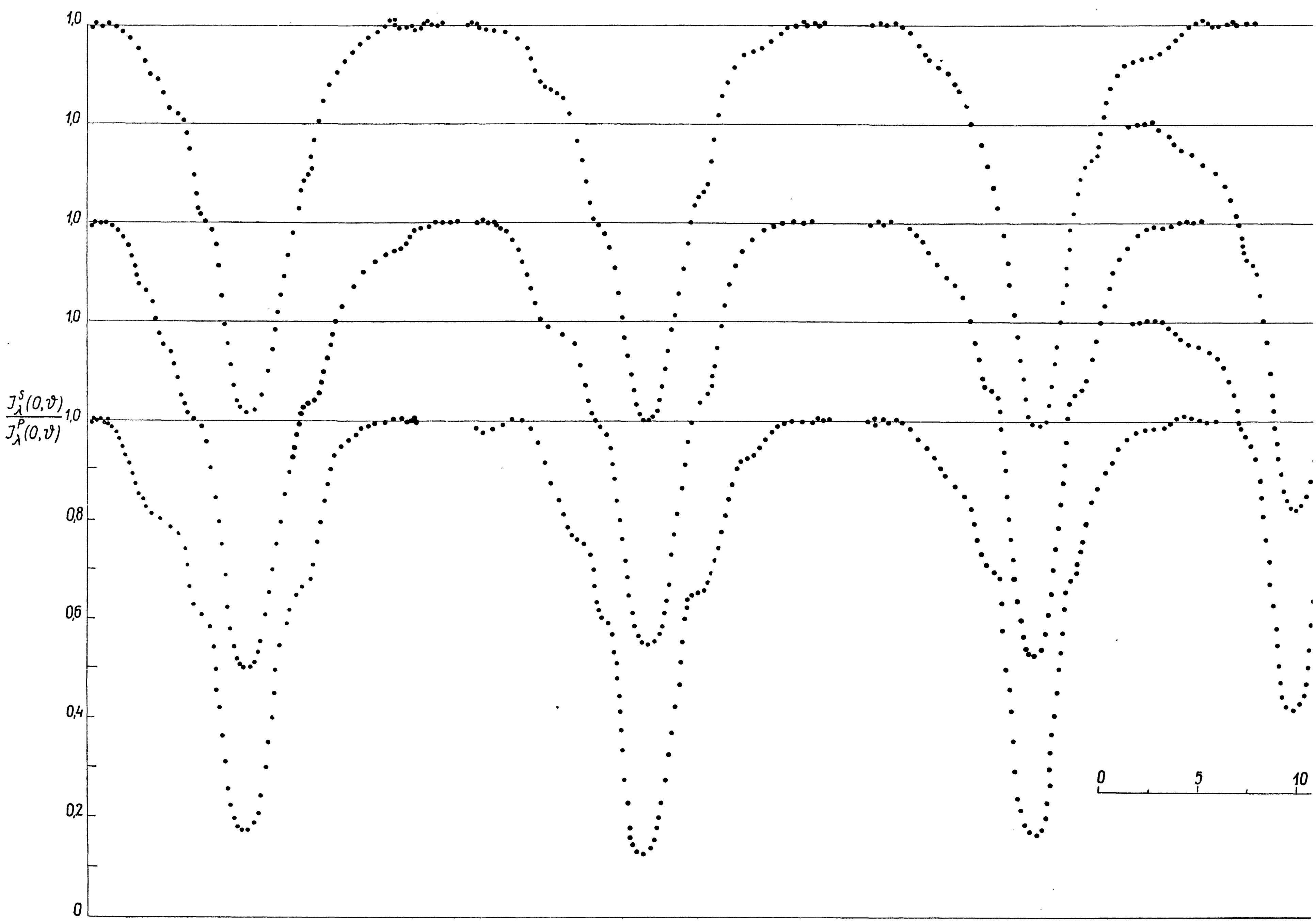

Fig. 2. Photometrical profiles for sunspot No 532.

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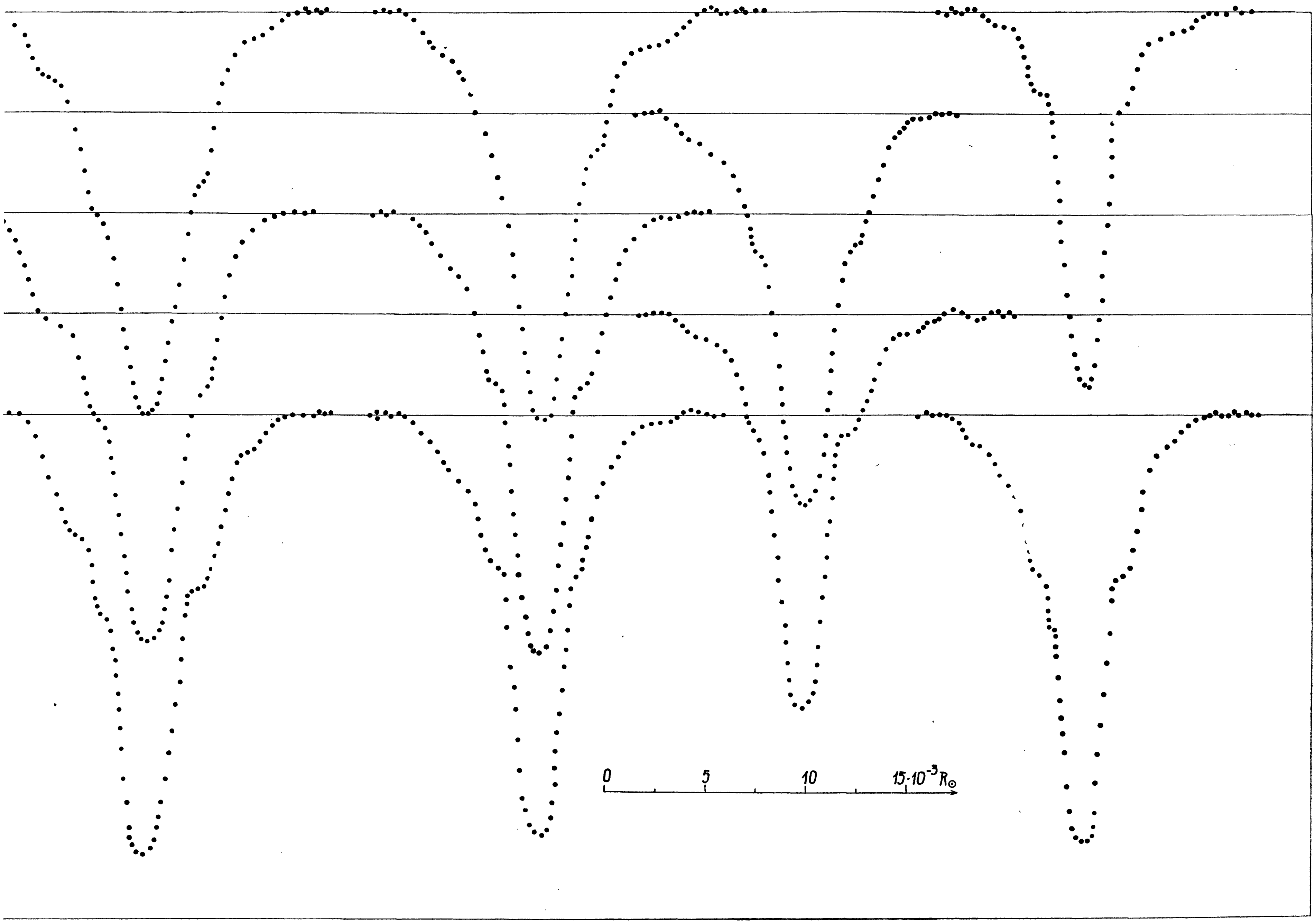

Fig. 2. Photometrical profiles for sunspot No 532.

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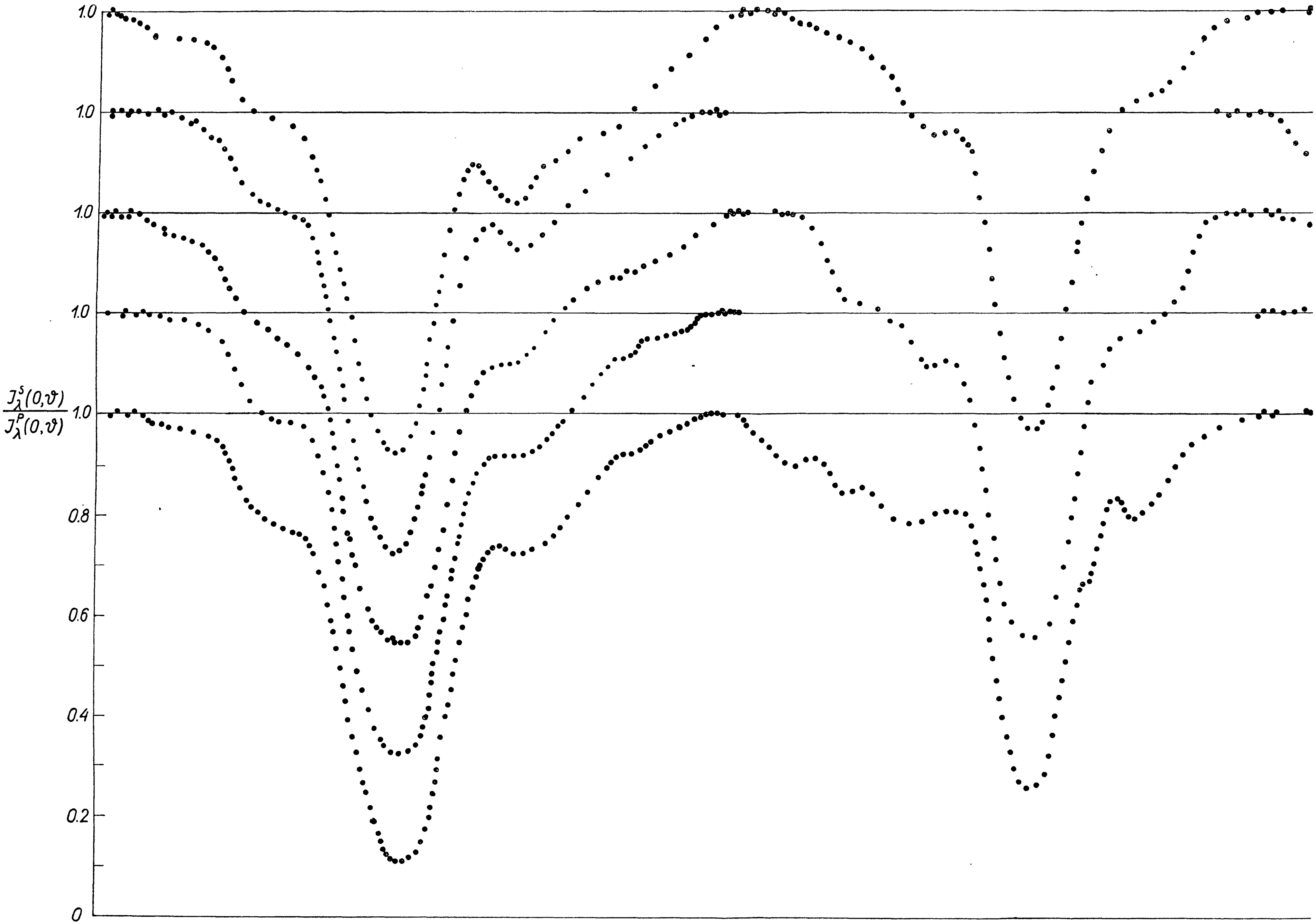

Fig. 3. Photometrical profiles for sunspot No 508.

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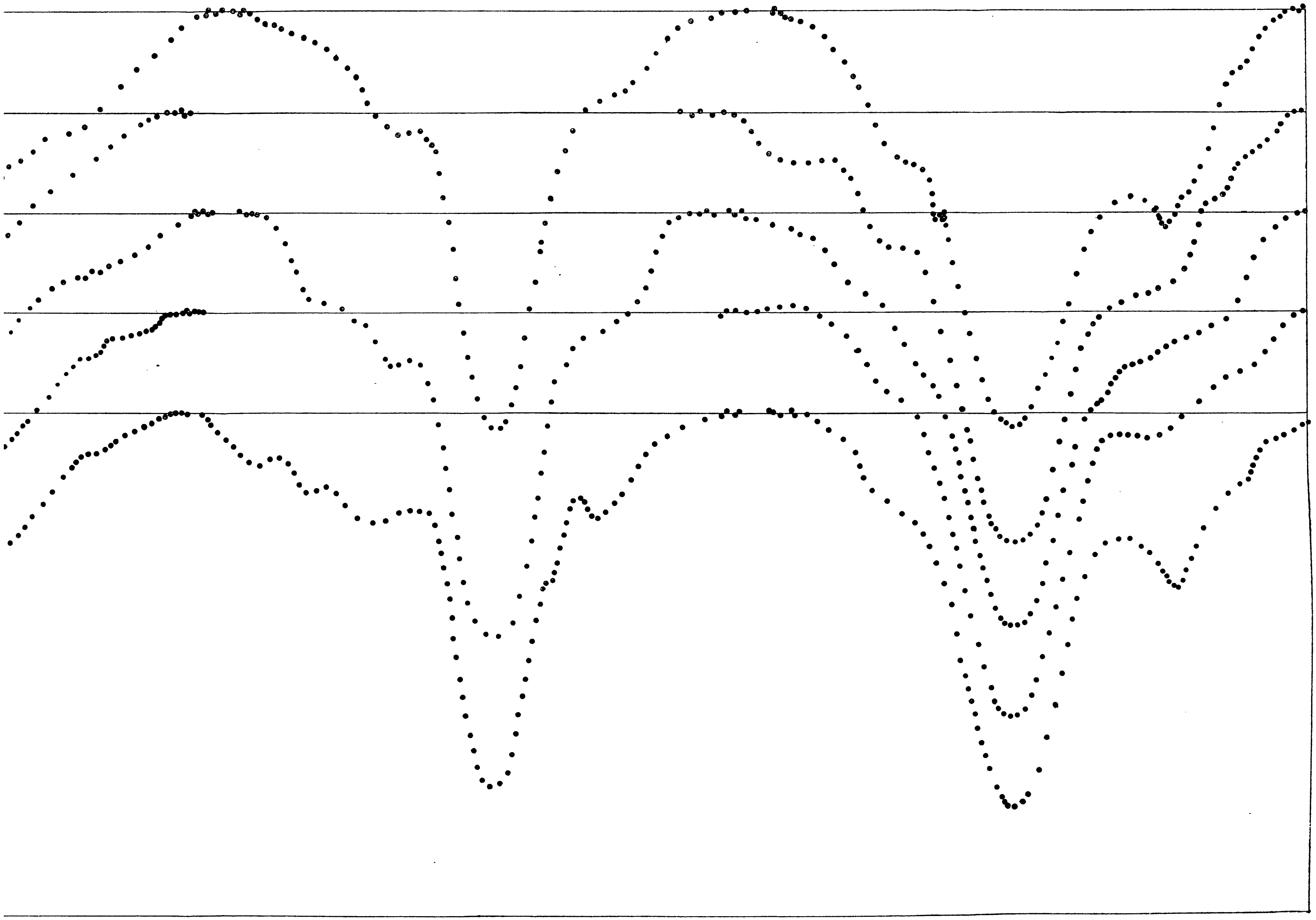

Fig. 3. Photometrical profiles for sunspot No 508.

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a correction for the atmospherical turbulence

$$\Delta'' = S\,(a, P)$$

was calculated by the method described by Stepanov [17]. In the preceding formula S denotes a function of the geometrical parameters

Table 1

Observed intensities of sunspots umbrae in units of the surrounding photosphere.

Day	Tokyo No	$\sin\vartheta$	λ(Å)	intensities $J^s_\lambda(0\vartheta)/J^p_\lambda(0\vartheta)$				
1958 Aug. 12	508	0.328	6305	0.170	0.210	0.130	0.150	0.140
			6172	0.180	0.190	0.120	0.140	0.145
			5596	0.120	0.139	0.090	0.100	0.112
			4500	0.076	0.054	0.070		
13	508	0.394	6305	0.175	0.200	0.220	0.135	0.130
			6172	0.165	0.190	0.200	0.130	0.125
14	508	0.529	6305	0.185	0.215	0.205	0.155	0.165
			6172	0.172	0.175	0.195	0.155	0.145
			5596	0.135	0.145	0.156	0.104	0.112
			5310	0.122	0.100	0.110	0.095	0.091
15	508	0.684	6305	0.210	0.170	0.205	0.185	0.145
			6172	0.215	0.180	0.190	0.160	0.155
			5596	0.138	0.146	0.105	0.115	0.120
			5310	0.122	0.114	0.091	0.100	0.110
			4500	0.084	0.082	0.080	0.070	0.065
16	508	0.819	6305	0.240	0.190	0.160	0.230	0.155
			6172	0.220	0.180	0.198	0.215	0.153
			5596	0.125	0.137	0.156	0.162	0.144
			4500	0.084	0.096	0.060	0.070	0.076
17	535	0.877	5596	0.200	0.225	0.210		
			4500	0.084	0.076	0.110		
	532	0.426	5596	0.200	0.190	0.195		
			4500	0.114	0.100	0.110		
18	535	0.761	5596	0.160	0.165	0.169		
			4500	0.085	0.095	0.100		
	532	0.324	5596	0.193	0.177	0.193		
			4500	0.096	0.100	0.120		
19	535	0.626	5596	0.170	0.160	0.165		
			5310	0.146	0.130	0.135		
			4500	0.090	0.099	0.087		
	532	0.381	5310	0.140	0.152	0.167		
			4500	0.099	0.114	0.120		
			6305	0.250	0.245	0.253		
20	535	0.477	6305	0.220	0.230	0.210		
			6172	0.215	0.235	0.230		
			5596	0.170	0.160	0.155		
			4500	0.090	0.095	0.102		
	532	0.522	6305	0.250	0.255	0.245		
			6172	0.230	0.235	0.225		
			5596	0.170	0.200	0.190		
			4500	0.125	0.100	0.116		
21	535	0.382	5596	0.175	0.150	0.166		
			5310	0.145	0.152	0.148		
			4500	0.096	0.087	0.099		
	532	0.671	5596	0.245	0.219	0.190		
			4500	0.135	0.114	0.100		
22	532	0.809	5596	0.215	0.190	0.247		
			4500	0.126	0.140	0.116		
24	535	0.652	6305	0.250	0.220	0.240		
			6175	0.257	0.230	0.220		
			5596	0.150	0.165	0.170		
			4500	0.110	0.124	0.107		
			5310	0.114	0.165	0.158		

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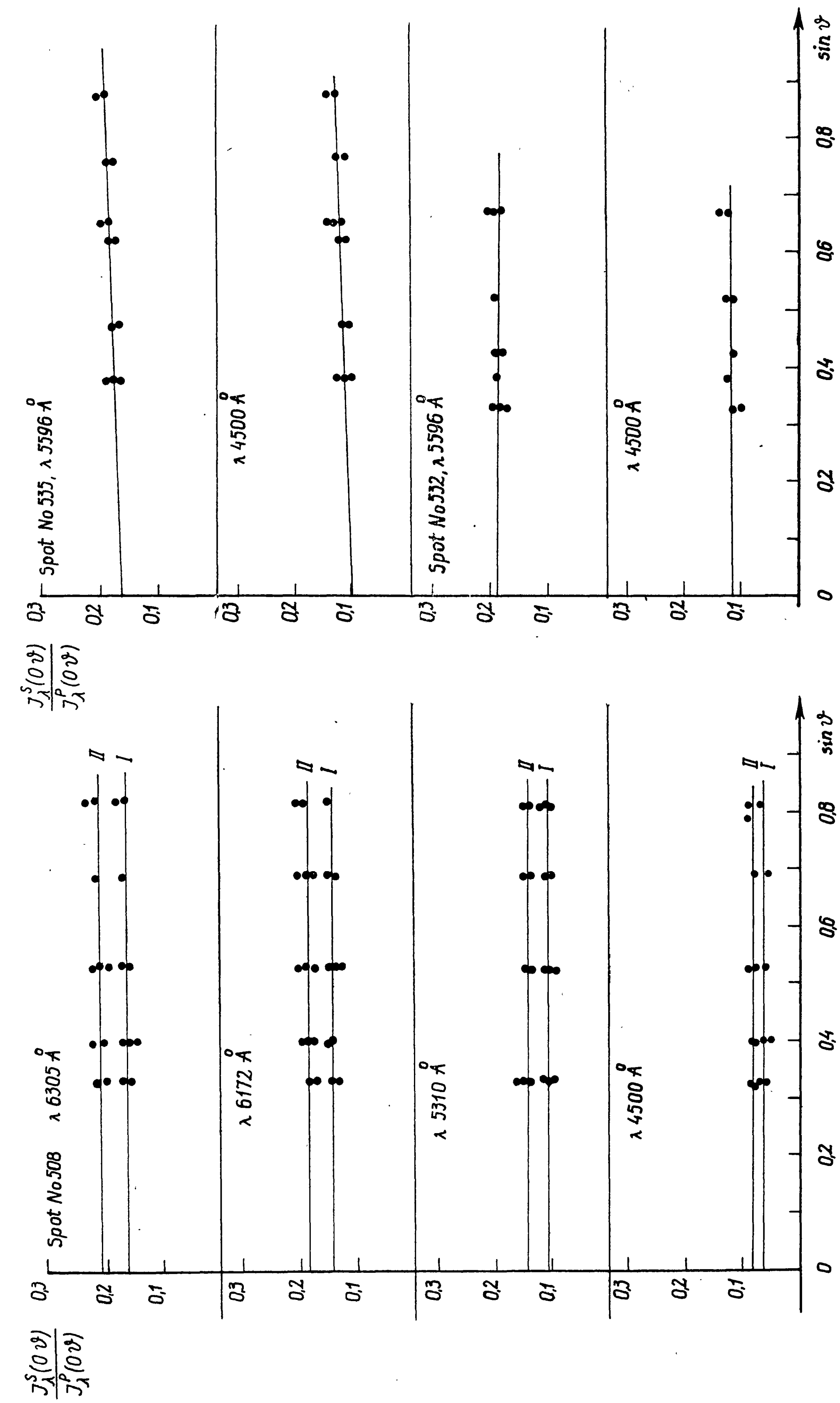

Fig. 4. Observed limb effect for the four sunspots.

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of the sunspot and of Wandens constant „a". This constant was calculated using the measurements of intensity near the sun's limb. The oscillations of the image were limited in our case to the interval $0''5 - 2''0$. The corrections $\triangle''$ calculated for the best atmospherical conditions had the values ca. 0·005 of the intensity of the photosphere. The light scattering did not exceed 3 percent of the intensity of the photosphere and it effected corrections to the central intensity of sunspots of an order of 10 to 30 percent. Examples of the observed photometrical profiles of sunspots are given in Figs. 1—3, and the corrected intensities of sunspots umbra in Fig. 4. In this figure we have, for each sunspot separately, the intensities as functions of sin ϑ in units of the intensity of the surrounding photosphere. The numeration of the sunspots given here was adopted from *Tokyo Bull. of Solar Phenom.* [4]. Considering the results mentioned above (Table 1) we found the intensity distribution in the continuous spectrum of the investigated sunspots.

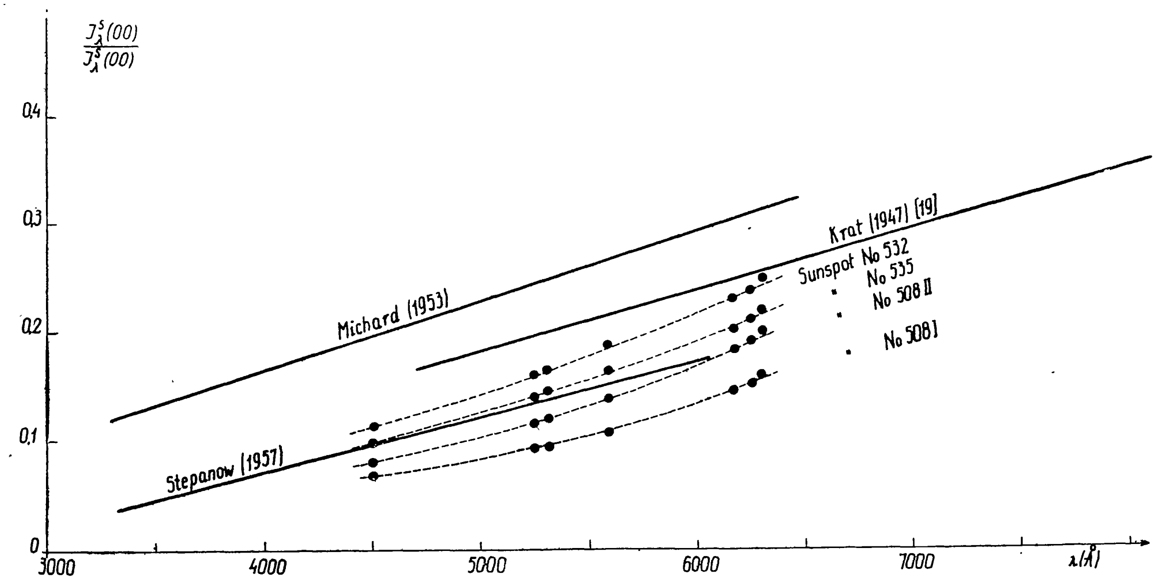

Fig. 5. Distributions of the continuous spectrum of sunspots.

Table 2

Relative intensities of the continuous spectrum of umbrae of sunspots.

Spot No	λ (Å) : 6305	6172	5596	5310	4500
535	0·221	0·214	0·165	0·146	0·099
532	0·250	0·230	0·190	0·167	0·114
508I	0·202	0·185	0·138	0·121	0·080
508II	0·160	0·146	0·108	0·094	0·068

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This distribution corresponds to sunspots placed in the center of the disc. The results are given in Fig. 5. and in Table 2. Fig. 5. represents the distribution of the relative intensities of the continuous spectrum of sunspots found from our observations as well as similar distributions given by other authors.

It results from an analysis of the measurement errors that plate calibration and photospheric intensity errors give rise to an error which does not exceed 0·030 and its mean value is

$$(I_\lambda^S / I_\lambda^P) = \pm 0{\cdot}015.$$

3. The determination of $\varkappa_\lambda/\bar{\varkappa}$ in sunspots

The relative intensities of the continuous spectrum of sunspots given in Table 2 may be evaluated in absolute units on the basis of the energy distribution of the continuous spectrum of the photosphere. The values of $I_\lambda^P(0,0)$ were adopted from Abbot's observations improved by Minnaert [12] and an interpolation was carried out for the desired wave lengths. The values $I_\lambda^P(0,0)$ mentioned above and the values $I_\lambda^S(0,0)$ calculated for the observed sunspots in units of

Table 3

Energy distribution in the continuous spectrum of the photosphere and of observed sunspots (in units of 10^{14} erg cm^{-2} sec^{-1}).

Spot No	λ (Å) : 6305	6172	5596	5310	4500
535	0·700	0·713	0·650	0·616	0·460
532	0·792	0·766	0·749	0·704	0·530
508I	0·640	0·616	0·544	0·515	0·372
508II	0·507	0·486	0·426	0·397	0·316
photo-sphere	3·17	3·33	3·94	4·22	4·65

10^{14} erg cm^{-2} sec^{-1} (for $\triangle\lambda = 1$ cm and $\triangle\Omega = 1$ sterad) are given in Table 3. For the obtained values of energy given in Table 3 we can calculate the temperatures of monochromatic radiation using Planck's formula. The results are given in Table 4.

The comparison of temperatures corresponding to various wave lengths indicates their small differences and hence we can assume a local thermodynamical equilibrium existing in the observed sunspots. The emerging intensity is in this case equal to

$$I^S(0\vartheta) = \int_0^\infty B_\lambda(T_s)\, e^{-\frac{\varkappa_\lambda}{\bar{\varkappa}}\tau \sec\vartheta}\, d\left(\frac{\varkappa_\lambda}{\bar{\varkappa}}\tau \sec\vartheta\right)$$

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Table 4

Monochromatic radiation temperatures of observed sunspots.

Sunspot No	λ (Å) : 6305	6172	5596	5310	4500
535	4440	4450	4420	4420	4410
532	4540	4510	4530	4520	4500
508I	4360	4340	4290	4300	4290
508II	4170	4155	4130	4125	4140

Making use of a solution given by Chandrasekhar and Breen [6] this formula may be rewritten in the form

$$I_\lambda^S(0,\vartheta) = \frac{2\,hc^2}{\lambda^5} \frac{1}{e^{\alpha\,(4/\sqrt{3})^{1/4}} - 1} J(\alpha, \beta)$$

where

$$\alpha = hc/k\lambda Te \quad ; \quad \beta = \frac{\varkappa_\lambda}{\bar{\varkappa}} \sec\vartheta$$

and

$$J(\alpha, \beta) = \int_0^\infty e^{-\beta\tau}\, b_\alpha(\tau)\, d\,\beta\tau$$

is a function tabulated by Chandrasekhar on the following two assumptions:

$$\text{(i)}\ \ T^4 = {}^3/_4\, Te^4\,(\tau + g\,(\tau))\ ;\ g\,(0) = 1/\sqrt{3} \qquad (*)$$

and (ii) $\varkappa_\lambda/\bar{\varkappa}$ does not depend upon the optical depth in the atmosphere. The calculations were carried out for the center of the disc where $\sec\vartheta = 1$ and hence the parameter β gives exactly the value of $\varkappa_\lambda/\bar{\varkappa}$. This procedure gives for each sunspot separately a system of 5 equations corresponding to the chosen wave lengths. From these equations we can evaluate the desired values of β for the prescribed values of Te. On the other hand the observations of the limb darkening of sunspots give us the temperature distributions as a function of optical depth for various wave lengths and in consequence the ratios of the corresponding absorption coefficients $\varkappa(\lambda_i)/\varkappa(\lambda_j)$. For the given values of the wave lengths we transform the function $J(\alpha, \beta)$ to the form $J_\lambda(Te, \beta)$ depending on (Te, β). Then for each observed sunspot we evaluate

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$$J_\lambda(Te,\beta)=\frac{[I_\lambda^S(0,0)]^{\text{obs.}}}{\left(\frac{2hc^2}{\lambda^5}\right):(e^{\alpha(4/\sqrt{3})^{1/4}}-1)}$$

for different values of Te. The effective temperature of the sunspots is not yet known. The procedure described above prescribes 5 values of the coefficient β to each value of Te.

$$\begin{array}{lll} Te^{(1)} & ; & \beta^{(1)}(\lambda_1)\ \dots\dots\dots\dots\ \beta^{(1)}(\lambda_5) \\ \vdots & & \dots\dots\dots\dots\dots\dots\dots\dots \\ Te^{(n)} & ; & \beta^{(n)}(\lambda_1)\ \dots\dots\dots\dots\ \beta^{(n)}(\lambda_5) \end{array}$$

If we know from observations of the limb darkening the ratio $\varkappa(\lambda_i)/\varkappa(\lambda_j)$ for at least two wave lengths from among five chosen ones we can determine the desired values of β by picking out from the calculated values of $\beta^{(n)}$ those which satisfy the condition

$$\varkappa(\lambda_i)/\varkappa(\lambda_j)=\beta^{(n)}(\lambda_i)/\beta^{(n)}(\lambda_j)$$

for some Te. This procedure yields an effective temperature and the corresponding coefficients β i. e. $\varkappa_\lambda/\bar{\varkappa}$:

$$Te\ ;\ \frac{\varkappa_1}{\bar{\varkappa}}\ \dots\dots\dots\dots\dots\dots\ \frac{\varkappa_5}{\bar{\varkappa}}$$

for each sunspot separately. As an example the results of the calculation of $J_\lambda(Te,\beta)$ and β are given in Fig. 6.

4. Temperature distribution

We get from observation the relative values $I_\lambda^S(0,\vartheta)/I_\lambda^P(0,\vartheta)$ of limb darkening of the sunspots (see Fig. 4.). By multiplying the obtained values by the corresponding values of the limb darkening of the photosphere and normalizing them to one we get for the sunspots:

$$\frac{J_\lambda^S(0,\vartheta)}{J_\lambda^S(0,0)}=\left(\frac{J_\lambda^S(0,\vartheta)}{J_\lambda^P(0,\vartheta)}\right)\left(\frac{J_\lambda^P(0,\vartheta)}{J_\lambda^P(0,0)}\right)\left(\frac{J_\lambda^S(0,0)}{J_\lambda^P(0,0)}\right)^{-1}$$

The interpolation of the values of the limb darkening of the photosphere was carried out by using Abbot's data [2] and the ratio $J_\lambda^S(0,0)/J_\lambda^P(0,0)$ was obtained by extrapolation of the observational curves $J_\lambda^S(0,\vartheta)/J_\lambda^P(0,\vartheta)$ for the values $\cos\vartheta=1$.
Assuming that the curve of the limb darkening is of the form (Chalonge [5],

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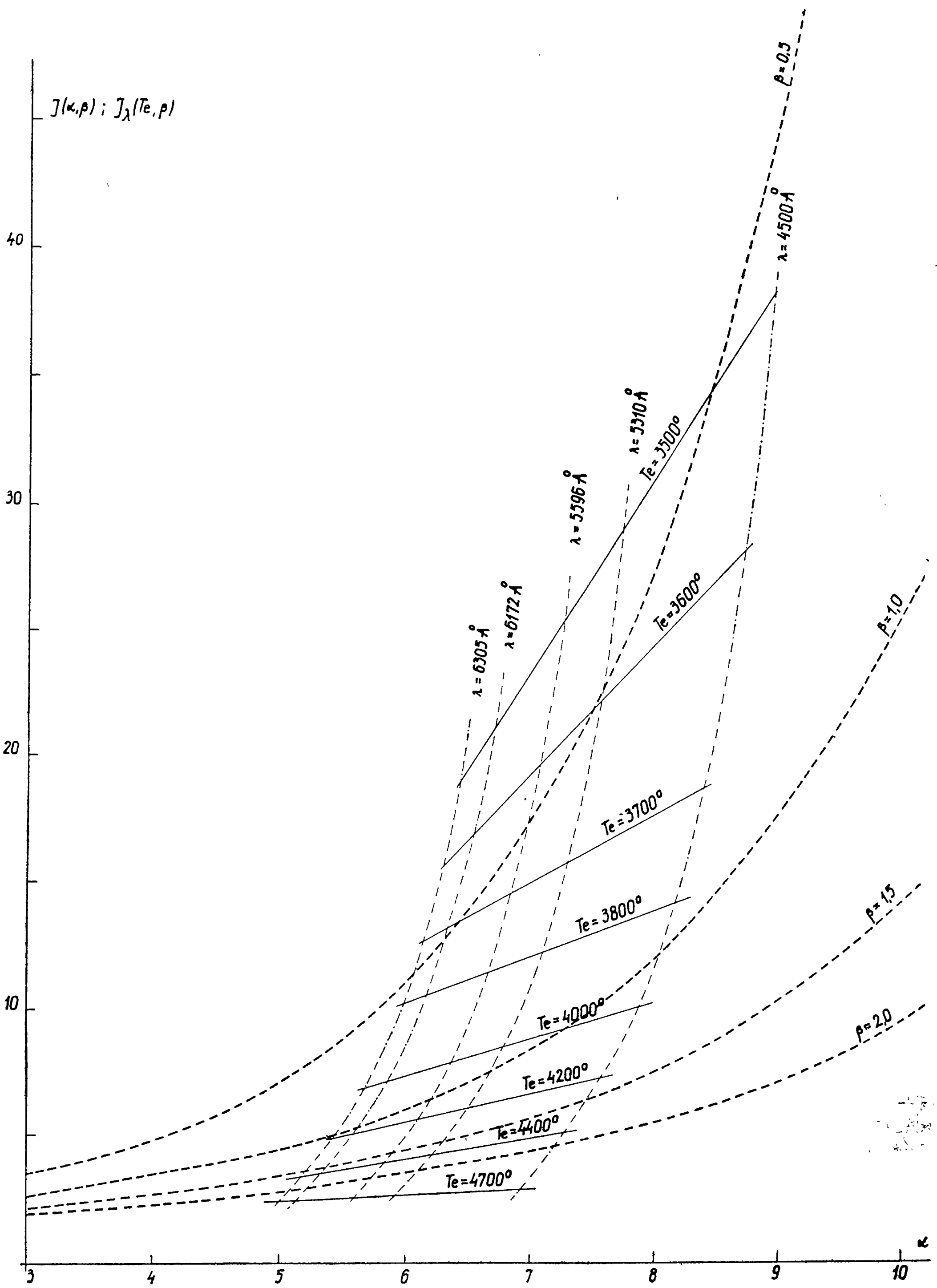

Fig. 6. The functions $J(\alpha, \beta)$ adopted from Chandrasekhar [6] and the values of $J_\lambda(Te, \beta)$ calculated for sunspot No 535.

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$$I_\lambda^S(0,\vartheta)/I_\lambda^S(0,0) = a_0 + a_1 \cos\vartheta + 2a_2\cos^2\vartheta$$

we evaluate the coefficients a_0, a_1, a_2 by the method of least squares. The system of equations

$$x_1 + x_2\cos\vartheta_i + x_3\cos^2\vartheta_i = l_i \qquad i = 1, 2, \dots\dots n, \qquad (**)$$

where $l_i = I_\lambda^S(0,\vartheta_i)/I_\lambda^S(0,0)$ are the values given by observation may be written in the form (so called „cracovian"-form)

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \begin{Bmatrix} 1 & 1 & \dots & 1 \\ \cos\vartheta_1 & \cos\vartheta_2 & \dots & \cos\vartheta_n \\ \cos^2\vartheta_1 & \cos^2\vartheta_2 & \dots & \cos^2\vartheta_n \end{Bmatrix} = \begin{Bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{Bmatrix}$$

or simpler

$$\mathbf{X}\cdot\tau\,\mathbf{a} = \mathbf{l}$$

Here **a** denotes a cracovian of the coefficients of the system (**)
We introduce now a cracovian of the coefficients enlarged by the column of l_n

$$\mathbf{a}' = \{\mathbf{a}\,\mathbf{l}\}$$

and we denote

$$\mathbf{a}' = \mathbf{A}$$

Cracovian **A** is symmetric so we can find its triangular „square root" **r** such that

$$\mathbf{r}^2 = \begin{Bmatrix} r_{11} & r_{21} & r_{31} & r_{41} \\ 0 & r_{22} & r_{32} & r_{42} \\ 0 & 0 & r_{33} & r_{43} \\ 0 & 0 & 0 & r_{44} \end{Bmatrix}^2 = \mathbf{A}$$

The equation which gives the values of the unknown variables yielding the smallest square deviation has the form

$$\mathbf{X}\,\tau\,\mathbf{r} = \mathbf{0}$$

as has been proved by T. Banachiewicz [3]. The method described above requires far less computing than the classical algorithm of Gauss and is much simpler too. It gives also an estimation of the error.

The coefficients of the limb darkening obtained in this way are presented in Table 5. To verify how the approximation constructed by means of the coefficients evaluated above works we calculated on their basis the limb darkening curves again and obtained a satisfactory agreement with the observational data. The differences O—C did not exceed 0·008 which lies in the limits of errors of observation.

It is seen therefore that the formula of Chalonge happened to be sufficiently accurate in our case and we did not need to use the formula of Kourganoff [9] where an exponential term occurs.

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Table 5

Coefficients of the limb darkening for the umbrae of observed sunspots.

Sunspot No	λ (Å)	a_0	a_1	a_2
535	5595	0·6057	0·7930	—0·2030
	4500	0·3131	1·1518	—0·2342
508, 532	6305	0·3845	0·7747	—0·0806
	6172	0·4122	0·6676	—0·0388
	5595	0·2992	0·9053	—0·1047
	5310	0·3234	0·7540	—0·0389
	4500	0·1332	1·1147	—0·1276

By using the formula of Chalonge we get the following solution of the transfer equation

$$B_\lambda (Ts)/I_\lambda^S (0,0) = a_0 + a_1 \tau + a_2 \tau^2$$

Having the values of $I_\lambda^S (0,0)$ we can find the distribution of temperature $T(\tau_\lambda)$. To compute $I_\lambda^S (0,0)$ we have to use the distribution of

Table 6

A comparison of temperature distributions $T(\tau_\lambda)$ computed with the intensity $I_\lambda^P(0,0)$ adopted from the data of Abbot (A) and of Minnaert (M).

	Sunspot No 535 λ 4500 Å		Sunspot No 535 λ 5596 Å	
τ_λ	$T(\tau_\lambda)(A)$	$T(\tau_\lambda)(M)$	$T(\tau_\lambda)(A)$	$T(\tau_\lambda)(M)$
0·00	3795	3790	4070	4020
0·05	3880	3865	4110	4060
0·1	3950	3930	4150	4100
0·2	4060	4040	4220	4170
0·3	4155	4135	4280	4230
0·4	4230	4210	4340	4280
0·5	4300	4280	4380	4330
0·6	4360	4340	4420	4370
0·7	4410	4390	4470	4410
0·8	4460	4440	4500	4450
0·9	4500	4480	4540	4480
1·0	4540	4520	4560	4510
1·2	4610	4590	4610	4550
1·4	4670	4640	4640	4580
1·6	4720	4700	4660	4600
1·8	4730	4710	4680	4610
2·0	4740	4730	4680	4620

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energy for the continuous spectrum of the photosphere. From various data given by various authors we adopted the values of $I_\lambda^P(0,0)$ from Abbot [12] and from Minnaert [13] to get $I_\lambda^S(0{\cdot}0)$. For these two lists of values of $I_\lambda^S(0,0)$ we calculated $T(\tau_\lambda)$. The results of calculations for sunspot No 535 are given in Table 6. It is seen that the differences are small and do not exceed 20—50° K. The distribution of the temperature for the remaining cases was computed by using Abbot's data for the energy distribution in the continuous spectrum of the photosphere. Once the distribution of temperature $T(\tau_\lambda)$ was evaluated we were able to find both the ratios of optical depth $\tau(\lambda_i)/\tau(\lambda_j)$ for layers with the same temperature and the corresponding ratios of the coefficients of absorption $\varkappa(\lambda_i)/\varkappa(\lambda_j)$. Following the procedure described in section 3, we get an exact agreement of the coefficients $\varkappa_\lambda/\bar{\varkappa}$ resulting from the distribution of the continuous spectrum of the sunspots with the ratios of the coefficients of absorption obtained by comparison of the distributions of $T(\tau_\lambda)$ for the values of effective temperatures given in Table 7.

Table 7

Effective temperatures and the coefficients of absorption $\varkappa_\lambda/\bar{\varkappa}$ obtained for the observed sunspots

Sunspot No	Te	($\varkappa_\lambda/\bar{\varkappa}$ ± 0·05)				
		λλ: 6305	6172	5596	5310	4500 Å
535	4420	1·47	1·48	1·55	1·59	1·70
532	4500	1·44	1·48	1·50	1·55	1·68
508I	4300	1·47	1·45	1·57	1·61	1·75
508II	4130	1·48	1·50	1·65	1·67	1·70

Having the values of $\varkappa_\lambda/\bar{\varkappa}$ we can reduce the distributions of $T(\tau_\lambda)$ to the wave length $\lambda = 5000$ Å. The distributions obtained in this way for 4 observed sunspots are given in Table 8 and in Fig. 7. For sunspots No 508 and No 532 the coefficients of the limb darkening had the same values as for the photosphere and the corresponding distributions had a similar form. For sunspot No 535 we got a smaller value for the limb darkening than the corresponding value for the photosphere and for this reason the gradient of the temperature is considerably smaller in this case than in the remaining cases. In addition in Fig. 7. the temperature distributions for two values of effective temperatures $Te = 4300$ and $Te = 4400°$ K computed by using formula (*) are given. We assumed function $q(\tau)$ to equal Chandrasekhar's IV-th approximation [6a] of Hopf's function and $\tau_\lambda = \frac{\varkappa_\lambda}{\bar{\varkappa}}\tau$ for

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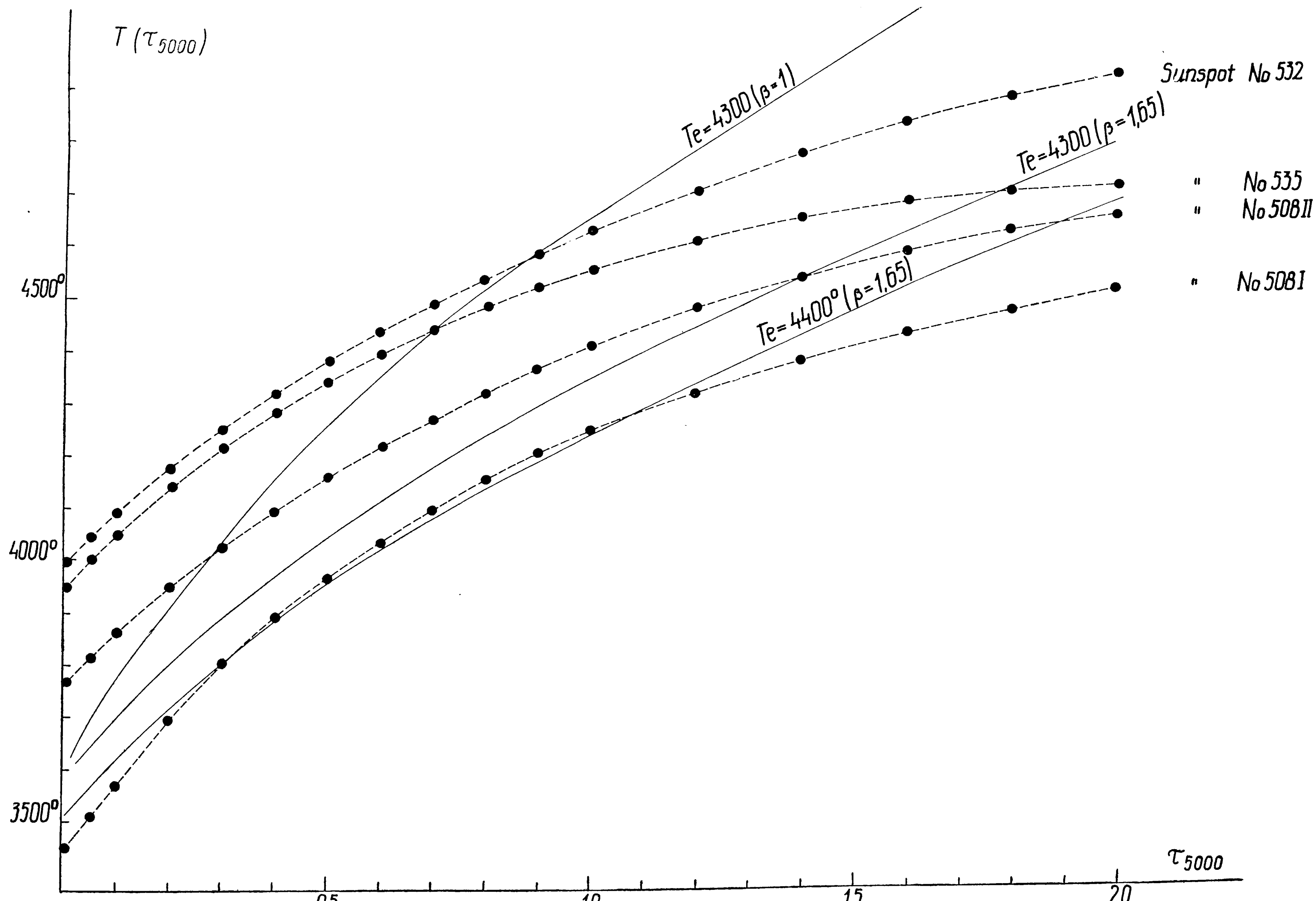

Fig. 7. Temperature distributions of the observed sunspots and the distributions calculated on the basis of Chandrasekhar's approximation.

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the $\varkappa_\lambda/\bar{\varkappa} = 1{\cdot}65$ (the value obtained with the data given in Table 7). As we can see in Fig. 7. the gradient of temperature accepted by Chandrasekhar is greater than the corresponding ones resulting from the observation of sunspots. By chosing suitable values of $\varkappa_\lambda/\bar{\varkappa}$ and Te we can get an agreement of the observed distribution with the one accepted by Chandrasekhar only in a small range of optical depth. The obtained distributions of temperature were compared with those given by other authors (Fig. 8.). It is interesting to compare

Table 8

Temperature distributions $T(\tau_{5000})$ of the observed sunspots.

τ_{5000} \ Spot No	532	535	508I	508II
0·00	3990	3950	3750	3440
0·05	4050	4010	3810	3500
0·1	4090	4060	3860	3560
0·2	4180	4140	3940	3680
0·3	4250	4220	4020	3790
0·4	4320	4290	4090	3870
0·5	4380	4350	4160	3950
0·6	4430	4400	4210	4030
0·7	4480	4440	4260	4080
0·8	4530	4480	4310	4130
0·9	4570	4520	4360	4180
1·0	4620	4550	4400	4230
1·2	4710	4610	4480	4310
1·4	4780	4650	4540	4380
1·6	4840	4690	4590	4430
1·8	4890	4700	4630	4480
2·0	4930	4710	4660	4520

them with the photoelectric measurements carried out by Makita and Morimoto [10] for a large umbra. They got $T(\tau_\lambda)$ with a considerably smaller slope than in our case for sunspot No 535 and in consequence with a greater discrepancy in comparison with Chandrasekhar's distribution. The values of temperatures determined in our case are in general smaller than the corresponding values given by other authors. The increase of temperature with optical depth is slower than that evaluated by Michard [11] but faster than that given by Japanese authors. The comparison made above has no deep physical meaning and indicates only the differences in the models of individual sunspots.

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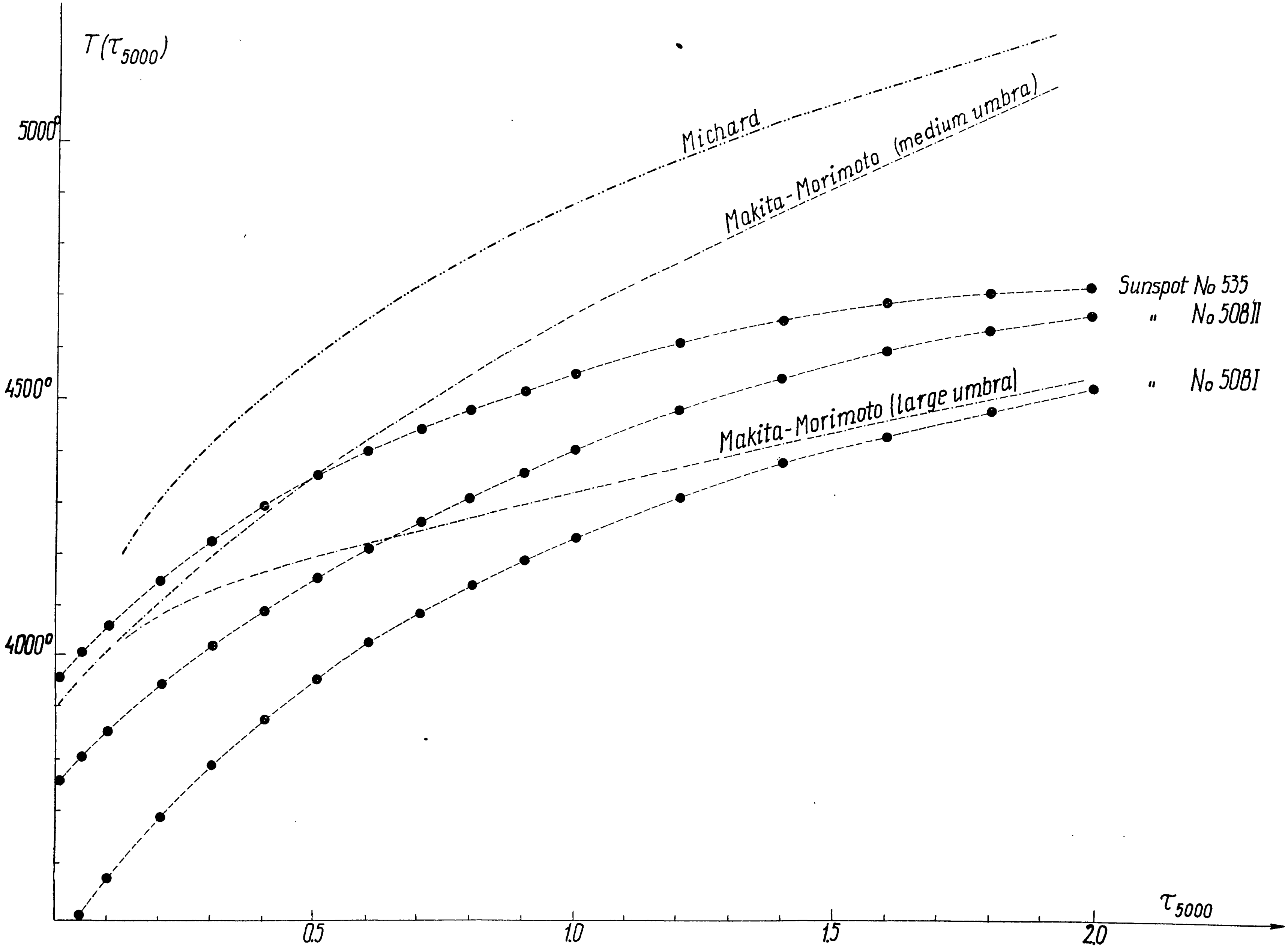

Fig. 8. The temperature distributions of the observed sunspots and the distributions given by M i c h a r d [11] and M a k i t a and M o r i m o t o [10]

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5. Conclusions

A determination of small differences in the intensities of a sunspot wandering along the Sun's disc is a very difficult problem but after all it may be solved by using the method of photographic photometry. It is necessary to keep in mind the fact that the relative measurements of limb darkening for the sunspots are charged with errors of observation which are much greater than the corresponding measurements for the photosphere. In addition the changes of the intensities effected by the evolution of sunspots may give rise to deformations of the limb darkening curve. The influence of these changes may be considerable in the early stages of development of sunspots. For sunspots No 532 and 535 the changes effected by the evolution are negligible. In the case of sunspot No 508 these changes are sufficiently great to produce an increased dispersion of the points of the limb darkening curve. On the other hand the computed empirical distributions of temperature depend on measurement data in a high degree. In particular even the small changes in a limb darkening curve may cause considerable changes in the computed distribution of temperature. In connection with that it is desirable to carry out the measurements of the intensity of sunspots with maximum accuracy. Many authors assume that the ratio of intensities of sunspot and photosphere is constant along the solar disc and in consequence the corresponding normalized limb darkening curves are identical. Even if we assume this, we can conclude from the analysis of our results that the sunspot cannot be treated as a photosphere with changed effective temperature. It is seen from Fig. 8 and 7 that the gradient of temperature describing the radiation of the photosphere sufficiently well cannot be used to describe the changes of temperature in a sunspot if we substitute the effective temperature of the Sun by the effective temperature of a sunspot in question. The differences will be much larger for sunspots which have a tendency to brightening for the observed intensities i. e. which have a smaller darkening than the photosphere. In addition a strong magnetic field may probably have a considerable influence on the energy conditions in sunspots. In this situation it seems necessary to analyze the applicability of the methods used in the investigations of the photosphere to questions connected with sunspots. When constructing a model of a sunspot it will be necessary to determine first the magnitude and the gradient of the magnetic field in the sunspot and then explain the connection of these values with the parameters of the model.

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Astronomical Institute
Wrocław, 1961. VI.

A. Stankiewicz

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